B. Sc. Part II (Hons) 4th Paper

Diff. Eqns (LDECC) (contd.)

Q. Solve $\dfrac{d^2y}{dx^2} + a^2y = \sec ax$.

Soln. For CF, $\dfrac{d^2}{dx^2} + a^2 = 0$

$\Rightarrow D^2 + a^2 = 0 \Rightarrow D = \pm ai$

$\therefore CF = c_1 e^{aix} + c_2 e^{-aix}$

$= c_1 (\cos ax + i \sin ax) + c_2 (\cos ax - i \sin ax)$

$= (c_1 + c_2) \cos ax + i (c_1 - c_2) \sin ax$

$= A \cos ax + B \sin ax$ where $A = c_1 + c_2$, $B = (c_1 - c_2) i$

For PI

$$PI = \frac{1}{D^2 + a^2} \sec ax$$

$$= \frac{1}{(D+ai)(D-ai)} \sec ax$$

$$= \frac{(D+ai) - (D-ai)}{2ai (D+ai)(D-ai)} \sec ax$$

$$= \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right] \sec ax$$

$$= \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right] \frac{e^{iax} \cdot e^{-iax}}{\cos ax} \quad \text{---(1)}$$

Now, $\frac{1}{D-ai} \frac{e^{iax} \cdot e^{-iax}}{\cos ax} = e^{iax} \cdot \frac{1}{D+ai-ai} \frac{e^{-iax}}{\cos ax}$

$= e^{iax} \cdot \frac{1}{D} \left[\frac{\cos ax - i \sin ax}{\cos ax} \right]$

$= e^{iax} \frac{1}{D} \left[1 - i \tan ax \right]$

$= e^{iax} \cdot \int (1 - i \tan ax) dx$

$\Rightarrow \frac{1}{D-ai} \sec ax = e^{iax} \left[x + \frac{i}{a} \log \cos ax \right]$

Similarly

$\frac{1}{D+ai} \sec ax = e^{-iax} \left[x - \frac{i}{a} \log \cos ax \right]$

So, (1) becomes,

$PI = \frac{1}{2ai} \left[e^{iax} \left(x + \frac{i}{a} \log \cos ax \right) - e^{-iax} \left(x - \frac{i}{a} \log \cos ax \right) \right]$

$\Rightarrow PI = \frac{1}{2ai} \left[x \left(e^{iax} - e^{-iax} \right) + \frac{i}{a} \log \cos ax \left(e^{iax} + e^{-iax} \right) \right]$

$= \frac{1}{2ai} \left[x \cdot 2i \sin ax + \frac{i}{a} \log \cos ax \cdot 2 \cos ax \right]$

$\Rightarrow PI = \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax \log \cos ax .$

Hence, the complete soln is given by

$y = CF + PI$

$\Rightarrow y = A \cos ax + B \sin ax + \frac{x}{a} \sin ax$

$+ \frac{1}{a^2} \cos ax \log \cos ax .$